In the special case $c = 0$ and $b \to \infty$ (pure intercept), we get

$$g_1 = \frac{k^3 T_{go}}{2 + k^2 T_{go}^2 - 2kT_{go} - 2\eta^{-1}} \tag{24}$$

$$g_2 = \frac{k^3 T_{go}^2}{2 + k^2 T_{go}^2 - 2kT_{go} - 2\eta^{-1}} \tag{25}$$

Using Eq. (11)

$$\dot{\lambda} = \frac{x_1 + x_2 T_{go}}{V_c T_{go}^2} \tag{26}$$

we obtain a new PN law (in the sequel referred to as a modified PN),

$$u = -N' V_c \dot{\lambda} \tag{27}$$

where

$$N' = \frac{k^3 T_{go}^3}{k^2 T_{go}^2 - 2kT_{go} + 2 - 2\eta^{-1}} \tag{28}$$

Two interesting properties of the new guidance law are

$$\lim_{T_{go} \to 0} N' = 3 \tag{29}$$

$$\lim_{k \to 0} N' = 3 \tag{30}$$

Note that $N' = 3$ matches the well-known PN law.

IV. Simulation Results

In this section, we consider a numerical example that illustrates the merits of the new guidance law. We analyze the effect on the trajectory of a 2-deg heading error. The end game is assumed to take 6 s, and the conflict is assumed to be a head on with a closing velocity of 3000 m/s. The effect of three guidance laws will be analyzed: 1) PN with $N' = 3$, 2) modified PN with $k = 0.3\ \mathrm{s}^{-1}$, and 3) modified PN with $k = 3\ \mathrm{s}^{-1}$.

Figure 2 shows the missile acceleration for the three different cases. The linear behavior with time of the acceleration under PN guidance is a well-known fact. Notice how the modified PN redistributes the acceleration along the flight. This is done by using higher PN gains at the earlier stages of the conflict as dictated by Eq. (28). The higher is k, the shorter is the accelerating time segment. This affects the trajectory as shown in Fig. 3.

V. Conclusions

A new PN guidance scheme was obtained with an exponentially weighted quadratic cost function. This modified PN, based on time-to-go estimates, redistributes the acceleration along the trajectory. The advantages of the new scheme become clear if we consider the fast reduction in maneuverability for aerodynamically maneuvering ground-to-air missiles. The derivation also provides the possibility to include weights on the terminal relative velocity in the cost function, thus producing more guidance schemes that are worth studying.

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Trigonometric Models for Large-Angle Aerodynamic Force Coefficients

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Introduction

NONTABULAR models of aerodynamic force coefficients used in mass-point characterizations of flight-vehicle motion are helpful for derivation and implementation of optimal guidance laws and predictions of vehicle performance. As interest develops in vehicle designs and tactics for large-α, large-β maneuvers, there is a corresponding increase in the need for aeroforce coefficient models valid in large-angle regimes. The force coefficients may be modeled in many ways to represent data from computational fluid dynamics work, wind-tunnel experiments, and/or flight sensors. Coefficient models may be tabular, analytic, or a blend of the two. By judicious use of analytic representations, the tabular storage of coefficients for large-angle dynamics can be greatly simplified without compromising accuracy, and the analyst can gain greater insight regarding force dependencies on the aeroangles (α and β).

Traditional aeroforce coefficient models are linear and quadratic relationships that are applicable over relatively limited angle ranges unless the parameters within these relationships are themselves varied with the aeroangles. The purpose of this Engineering Note is to illustrate simple steps whereby the aeroangle dependencies may be expressed analytically to make them explicit for both small and large angles. These steps can facilitate analytical treatments and lessen the complexity of stored tables or polynomial representations. The goal is for the parameters in the coefficient models to exhibit dependence only on nonangular variables such as Mach number, altitude, etc.

Classical models for aeroforce coefficients, written in terms of the American National Standards Institute/AIAA air-path (wind-axis) coordinate system are[1,2]

$$C_{D_W} = C_{D_0} + C_{L_W}^2 \big/ [\pi e (\mathcal{R})] \tag{1}$$

$$C_{Y_W} = C_{Y_\beta}\beta \quad \left(\text{nominally } C_{Y_\beta} < 0\right) \tag{2}$$

$$C_{L_W} = C_{L_\alpha}\alpha \tag{3}$$

where α is the angle of attack, defined as $\alpha = \tan^{-1}(w/u)$, and β is the angle of sideslip, defined as $\beta = \sin^{-1}(v/V)$, where u, v, and w are the linear velocity components along the x, y, and z body axes of the vehicle, respectively, whereas V is the resultant of the velocity components. C_{D_W} is the wind-axis drag coefficient. C_{D_0} is the sum of pressure and skin-friction drag coefficients; it is often referred to as the zero-lift drag coefficient. C_{Y_W} and C_{L_W} are the side-force and lift coefficients, respectively. ($\mathcal{R}$) is the wing planform aspect ratio, and e is the span efficiency factor. C_{Y_β} and C_{L_α} are the side-force derivative and lift-curve slope, respectively. Commonly, C_{D_0}, e, C_{Y_β}, and C_{L_α} are stored as tabular functions of α, β, Mach number, and altitude. The aerocoefficients may be defined for trimmed flight conditions, but often the control-surface deflections are treated as further independent variables.

A number of candidate analytic forms are available for representing aeroforce coefficients; these include regression curve fits using polynomial, exponential, or spline basis functions. Simple

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trigonometric functions are highly useful, and weighted products and/or sums of sines and cosines could also be employed. This Note briefly explores use of simple notional trigonometric models that are potentially suitable over aeroangle ranges of $-\pi/2$–$\pi/2$ for symmetrical shapes, or $-\pi/2$–0 and separately 0–$\pi/2$ for asymmetrical shapes.

Notional Body-Axis Force Coefficients

Suppose the body-axis aerodynamic force coefficients for trimmed flight of a vehicle are accurately expressed as follows:

$$C_A = a\cos\alpha\cos\beta \tag{4}$$

$$C_Y = b\cos\alpha\sin\beta \quad \text{(nominally } b < 0\text{)} \tag{5}$$

$$C_N = c\sin\alpha\cos\beta \tag{6}$$

in which a, b, and c are dimensionless. C_A refers to axial force, defined to be positive along the body negative x axis; C_Y refers to side (lateral) force, positive along the body y axis; and C_N refers to normal force, positive along the body negative z axis. Additional factors and terms could be used in Eqs. (4–6) if needed to reduce or eliminate α and β dependencies of a, b, and c. No assertion of generality is made for Eqs. (4–6); they are notional relationships used here to illustrate the suggested approach.

Transforming to Stability and Wind Axes

The intermediate (stability) axis transformations of Eqs. (4–6) are[1]

$$C_{D_S} = C_A\cos\alpha + C_N\sin\alpha \tag{7}$$

$$C_{Y_S} = C_Y \tag{8}$$

$$C_{L_S} = -C_A\sin\alpha + C_N\cos\alpha \tag{9}$$

which become, on substitution of Eqs. (4–6),

$$C_{D_S} = (a\cos^2\alpha + c\sin^2\alpha)\cos\beta \tag{10}$$

$$C_{Y_S} = b\cos\alpha\sin\beta \tag{11}$$

$$C_{L_S} = \tfrac{1}{2}(c-a)\sin 2\alpha\cos\beta \tag{12}$$

The airpath (wind) axis transformations of C_A, C_Y, and C_N are[1]

$$C_{D_W} = C_A\cos\alpha\cos\beta - C_Y\sin\beta + C_N\sin\alpha\cos\beta \tag{13}$$

$$C_{Y_W} = C_A\cos\alpha\sin\beta + C_Y\cos\beta + C_N\sin\alpha\sin\beta \tag{14}$$

$$C_{L_W} = -C_A\sin\alpha + C_N\cos\alpha \tag{15}$$

or, on substitution of Eqs. (4–6),

$$C_{D_W} = (a\cos^2\alpha + c\sin^2\alpha)\cos^2\beta - b\cos\alpha\sin^2\beta \tag{16}$$

$$C_{Y_W} = \tfrac{1}{2}(a\cos^2\alpha + b\cos\alpha + c\sin^2\alpha)\sin 2\beta \tag{17}$$

$$C_{L_W} = \tfrac{1}{2}(c-a)\sin 2\alpha\cos\beta \tag{18}$$

Note that it is readily shown that, if α is small and $\beta = 0$,

$$\alpha_{\left(C_{L_W}/C_{D_W}\right)_{\max}} = [a/(c-a)]^{\frac{1}{2}}$$

$$\alpha_{\left(C_{L_W}^{\frac{3}{2}}/C_{D_W}\right)_{\max}} = [3a/(c-a)]^{\frac{1}{2}}$$

These angles of attack produce maximum steady-state glide range and maximum steady-state glide endurance, respectively. Corresponding values for large α and large β may be found numerically from Eqs. (16–18).

Numerical Values of Coefficients for Notional Application

Consider the case of a notional gravity bomb that is steerable via onboard or remote-command guidance. Essentially a body of revolution, the bomb produces side and normal forces via the mechanism of crossflow drag, which is sensitive to body-surface roughness and Reynolds number. Suppose $a = 0.1$, $b = -2.5$, and $c = 2.5$; for these parameter values Figs. 1–4 present graphs of Eqs. (16–18) that present the following coefficients and ratios: Fig. 1, large-αC_{D_W} vs α for $\beta = 0$, 10, and 20 deg; Fig. 2, large-αC_{L_W} vs α for $\beta = 0$, 10, and 20 deg; Fig. 3, large-$\alpha C_{L_W}/C_{D_W}$ vs α for $\beta = 0$, 10, and 20 deg; and Fig. 4, large-αC_{D_W} vs C_{L_W} for $\beta = 0$, 10, and 20 deg.

The C_{Y_W} vs β curves for nonzero α (not presented here because of space limitations) are not identical to the C_{L_W} vs α curves for nonzero β. A given magnitude of α impairs C_{Y_W} more than the same magnitude of β impairs C_{L_W}. This peculiarity arises because of dissimilar (but standard) conventions used to define α and β, which make it appear preferable to maneuver primarily with α, rolling the vehicle to keep sideslip zero. In fact, an axisymmetric body of

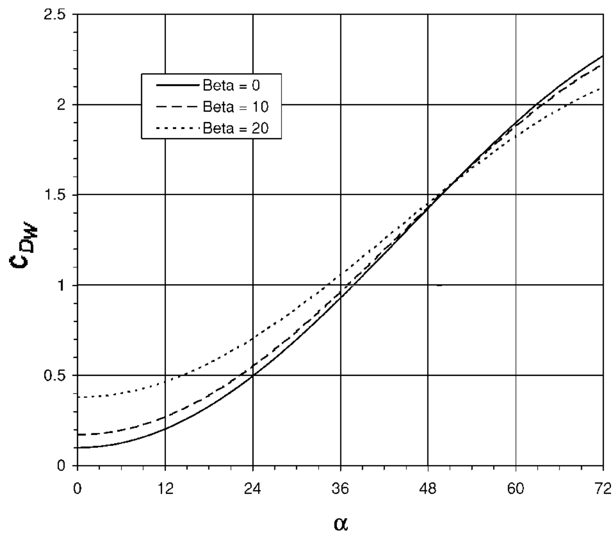

Fig. 1 Large-α drag coefficient vs α for $\beta = 0$, 10, and 20 deg.

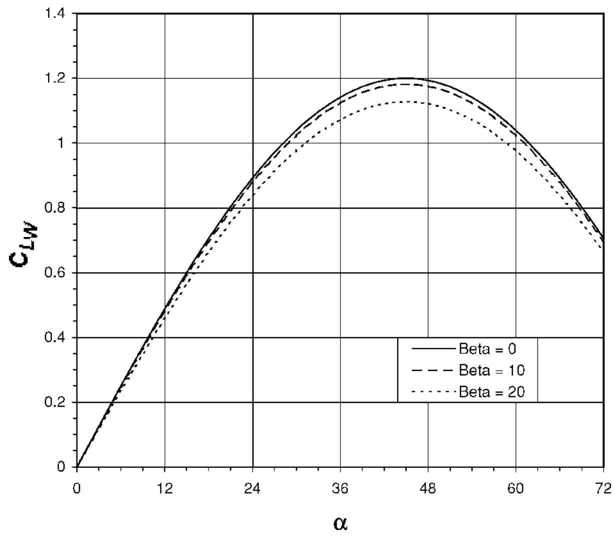

Fig. 2 Large-α lift coefficient vs α for $\beta = 0$, 10, and 20 deg.

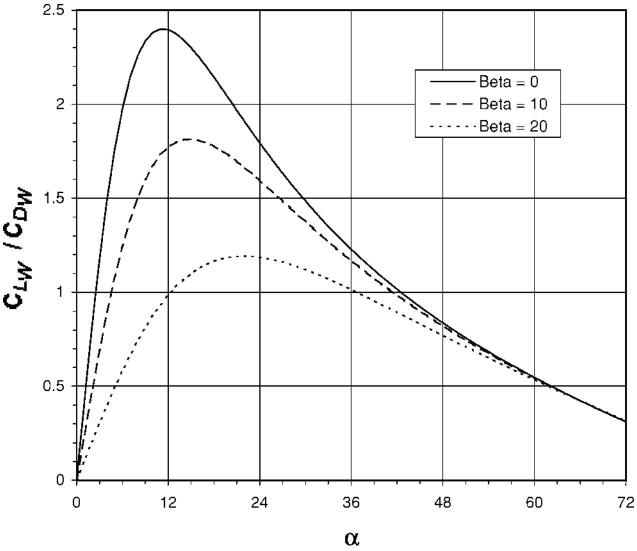

Fig. 3 Large-α lift-to-drag ratio vs α for $\beta = 0$, 10, and 20 deg.

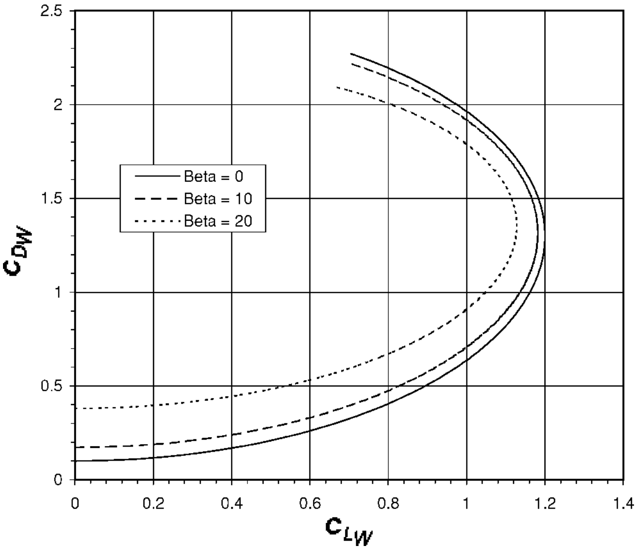

Fig. 4 Large-α drag polar for $\beta = 0$, 10, and 20 deg.

revolution is guided with equal effectiveness via a bank-to-turn or yaw-to-turn strategy.

Conclusions

Aeroforce coefficients for large α and β may be modeled via trigonometric functions that provide analytical insight and computational efficiency, benefit derivation of guidance laws, and could aid predictions of flight vehicle performance, particularly for aircraft and missiles that are maneuverable at large aeroangles.

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Two-Timescale Analysis of Phugoid Mode

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Introduction

SINCE the first Phugoid approximation derived by Lanchester,[1] literal approximations of aircraft modes have been of great interest to the aeronautical community. Classical approximations based on the hypothesis of complete and partial separation between fast and slow modes are well described by McRuer et al.[2] Earlier attempts to describe longitudinal modes based on singular perturbation theory have been proposed by Xu[3] and Khalil and Chen.[4] A recent approximation of longitudinal modes was described by Kamesh and Pradeep,[5,6] while further approximations based on the difference between the two timescales are given by Ananthkrishnan and Unnikrishnan[7] and Ananthkrishnan and Ramadevi.[8]

The reason why literal approximations of longitudinal modes are still of interest to flight dynamicists[9,10] can be found in the fact that while short-period parameters and Phugoid frequency are well estimated by classical simplified models, Phugoid damping approximation can still be improved.

In the present work we present an analytical procedure for Phugoid-mode literal approximation by applying different timescale analysis techniques. With respect to classical approximations, no assumption on separation between aircraft modes is made, and under the hypothesis of two-timescales property a change of variable is introduced, allowing the block diagonalization of the system into two slow and fast subsystems. The slow subsystem provides the new Phugoid-mode approximation. Numerical results show that the classical approximations of Phugoid damping are strongly improved.

The rest of the paper is organized as follows. First, the linearized aircraft dynamics are summarized and the proposed approximation based on the two-timescale technique is developed. Then, considerations on numerical results and conclusions are presented.

Longitudinal Aircraft Dynamics

For the purposes of this paper, we refer to a linearized set of equations about a steady flight trim condition. It is assumed that 1) $X_{\dot{w}}$, X_q, $Z_{\dot{w}}$, and Z_q stability derivatives are zero; and 2) the flight path of the aircraft is horizontal: $\gamma_0 = 0$. Under the preceding assumptions, the longitudinal equations of motions referred to a stability axis reference frame are

$$\dot{u} = X_u u - g\theta + X_w w, \qquad \dot{\theta} = q$$

$$\dot{w} = Z_u u + Z_w w + U_0 q$$

$$\dot{q} = M_u u + M_w w + M_{\dot{w}} \dot{w} + M_q q \tag{1}$$

Using standard notations,[2] the general polynomial form for the longitudinal aircraft dynamics is

$$\begin{aligned}\Delta_{\text{long}} &= As^4 + Bs^3 + Cs^2 + Ds + E \\ &= \left(s^2 + 2\xi_{\text{sp}}\omega_{\text{sp}}s + \omega_{\text{sp}}^2\right)\left(s^2 + 2\xi_p\omega_p s + \omega_p^2\right)\end{aligned} \tag{2}$$

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